Deriving the input file format of 8-point DCT function

The total derivation process is adopted from the following books:

Anirban Sengupta "Frontiers in Securing IP Cores - Forensic detective control and obfuscation techniques", The Institute of Engineering and Technology (IET), 2020, ISBN-10: 1-83953-031-6, ISBN-13: 978-1-83953-031-9

Anirban Sengupta, Saraju P. Mohanty "IP Core Protection and Hardware-Assisted Security for Consumer Electronics", The Institute of Engineering and Technology (IET), 2019, Book ISBN: 978-1-78561-799-7, e-ISBN: 978-1-78561-800-0

The generic equation of forward DCT can be expressed as (Obukhov et al., 2008):

$$X(m) = u(m)\sqrt{\frac{2}{N}}\sum_{i=0}^{N-1} x(i)\cos\left[\frac{(2i+1)m\pi}{2N}\right] \qquad (1)$$

In the above expression,

$$u(m) = \begin{cases} \frac{1}{\sqrt{2}}; for\ m = 0 \\ 1\ \ ; for\ m \neq 0 \end{cases}; \text{m = 0, 1, ..N-1}$$

x(i) is the input signal, X(m) is the output signal and N indicates number of data points. Therefore, for N=8 the first output signal X(0) can be expressed as follows:

$$X(0) = \frac{1}{\sqrt{2}}\sqrt{\frac{2}{8}}\sum_{i=0}^{7} x(i)\cos[0]$$

$$= \frac{1}{\sqrt{8}}\,x(0) + \frac{1}{\sqrt{8}}\,x(1) + \frac{1}{\sqrt{8}}\,x(2) + \frac{1}{\sqrt{8}}\,x(3) + \frac{1}{\sqrt{8}}\,x(4) + \frac{1}{\sqrt{8}}\,x(5) + \frac{1}{\sqrt{8}}\,x(6) + \frac{1}{\sqrt{8}}\,x(7)$$

$$= 0.3536\,x(0) + 0.3536\,x(1) + 0.3536\,x(2) + 0.3536\,x(3) + 0.3536\,x(4) + 0.3536\,x(5) + 0.3536\,x(6) + 0.3536\,x(7)$$

Similarly, for the same data point X(1) and X(2) can be expressed as:

$$X(1) = 1\sqrt{\frac{2}{8}}\sum_{i=0}^{7} x(i)\cos\left[\frac{(2i+1)1\pi}{16}\right]$$

$$= \frac{1}{2}\,x(0)\cos\left[\frac{\pi}{16}\right] + \frac{1}{2}\,x(1)\cos\left[\frac{3\pi}{16}\right] + \frac{1}{2}\,x(2)\cos\left[\frac{5\pi}{16}\right] + \frac{1}{2}\,x(3)\cos\left[\frac{7\pi}{16}\right] + \frac{1}{2}\,x(4)\cos\left[\frac{9\pi}{16}\right] + \frac{1}{2}\,x(5)\cos\left[\frac{11\pi}{16}\right] + \frac{1}{2}\,x(6)\cos\left[\frac{13\pi}{16}\right] + \frac{1}{2}\,x(7)\cos\left[\frac{15\pi}{16}\right]$$

$$= 0.4904\, x(0) + 0.4157\, x(1) + 0.2778\, x(2) + 0.0975\, x(3) + (-0.0975)\, x(4) + (-0.2778)\, x(5) + (-0.4157)\, x(6) + (-0.4904)\, x(7)$$

$$X(2) = 1\sqrt{\frac{2}{8}}\sum_{i=0}^{7} x(i)\cos\left[\frac{(2i+1)2\pi}{16}\right]$$

$$= \frac{1}{2}\, x(0)\cos\left[\frac{2\pi}{16}\right] + \frac{1}{2}\, x(1)\cos\left[\frac{6\pi}{16}\right] + \frac{1}{2}\, x(2)\cos\left[\frac{10\pi}{16}\right] + \frac{1}{2}\, x(3)\cos\left[\frac{14\pi}{16}\right] + \frac{1}{2}\, x(4)\cos\left[\frac{18\pi}{16}\right] + \frac{1}{2}\, x(5)\cos\left[\frac{22\pi}{16}\right] + \frac{1}{2}\, x(6)\cos\left[\frac{26\pi}{16}\right] + \frac{1}{2}\, x(7)\cos\left[\frac{30\pi}{16}\right]$$

$$= 0.4619\, x(0) + 0.1913\, x(1) + (-0.1913)\, x(2) + (-0.4619)\, x(3) + (-0.4619)\, x(4) + (-0.1913)\, x(5) + 0.1913\, x(6) + 0.4619\, x(7)$$

Similarly, X(3) to X(7) can be calculated using Eqn. (1).

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0.4904 & 0.4157 & 0.2778 & 0.0975 & -0.0975 & -0.2778 & -0.4157 & -0.4904 \\ 0.4619 & 0.1913 & -0.1913 & -0.4619 & -0.4619 & -0.1913 & 0.1913 & 0.4619 \\ 0.4157 & -0.0975 & -0.4904 & -0.2778 & 0.2778 & 0.4904 & 0.0975 & -0.4157 \\ 0.3536 & -0.3536 & -0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & 0.3536 \\ 0.2778 & -0.4904 & 0.0975 & 0.4157 & -0.4157 & -0.0975 & 0.4904 & -0.2778 \\ 0.1913 & -0.4619 & 0.4619 & -0.1913 & -0.1913 & 0.4619 & -0.4619 & 0.1913 \\ 0.0975 & -0.2778 & 0.4157 & -0.4904 & 0.4904 & -0.4157 & 0.2778 & -0.0975 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix}$$

Fig.1 Output signal of 8 point DCT in the form of matrix multiplication

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} C_4 & C_4 & C_4 & C_4 & C_4 & C_4 & C_4 & C_4 \\ C_1 & C_3 & C_5 & C_7 & -C_7 & -C_5 & -C_3 & -C_1 \\ C_2 & C_6 & -C_6 & -C_2 & -C_2 & -C_6 & -C_6 & C_2 \\ C_3 & -C_7 & -C_1 & -C_5 & C_5 & C_1 & C_7 & -C_3 \\ C_4 & -C_4 & -C_4 & C_4 & C_4 & -C_4 & -C_4 & C_4 \\ C_5 & -C_1 & C_7 & C_3 & -C_3 & -C_7 & C_1 & -C_5 \\ C_6 & -C_2 & C_2 & -C_6 & -C_6 & C_2 & -C_2 & C_6 \\ C_7 & -C_5 & C_3 & -C_1 & C_1 & -C_3 & C_5 & -C_7 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix}$$

Fig. 2. Another representation of output signal of 8 point DCT in the form of matrix multiplication

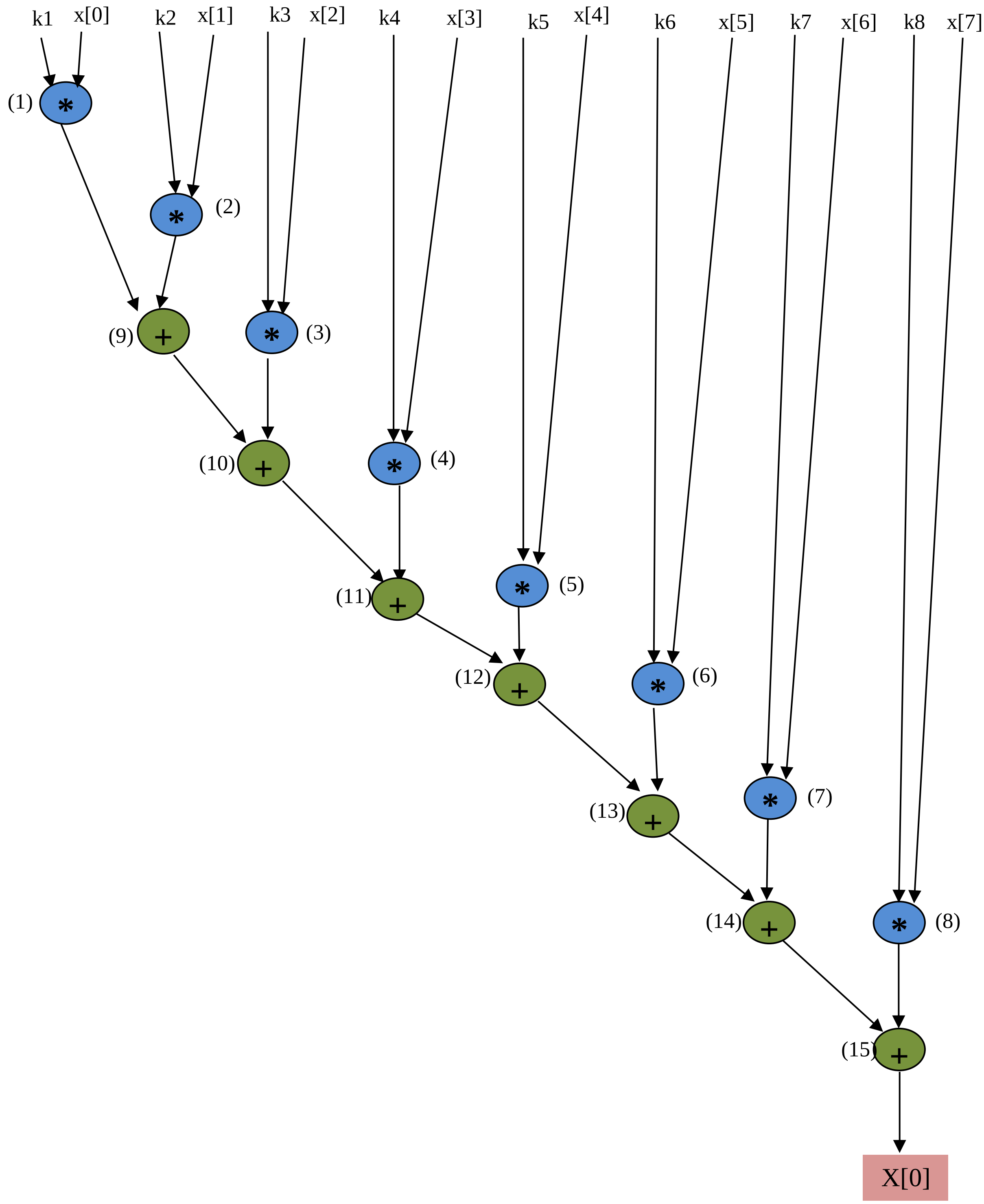

Fig. 3.Data flow graph of an 8-point DCT

We can represent this calculation in the form of matrix multiplication as shown in Fig. 1 where the 8x1 output signal matrix can be derived by multiplying 8 point DCT coefficient matrix with 8x1 input signal matrix. The 8 point DCT coefficient can also be represented as shown in Fig. 2, where C_1 to C_7 indicates the DCT coefficients in ascending order (C_1 indicates the maximum positive DCT coefficient value and C_7 indicates the minimum positive DCT coefficient value). Based on matrix multiplication as shown in Fig. 2, the output signals X[0] to X[7] can be represented as:

X[0]=c4*x[0]+c4*x[1]+c4*x[2]+c4*x[3]+c4*x[4]+ c4*x[5]+ c4*x[6]+ c4*x[7]

X[1]=c1*x[0]+c3*x[1]+c5*x[2]+c7*x[3]+(-c7)*x[4]+(- c5)*x[5]+ (-c3)*x[6]+ (-c1)*x[7]

X[2]=c2*x[0]+c6*x[1]+(-c6)*x[2]+(-c2)*x[3]+(-c2)*x[4]+(- c6)*x[5]+ c6*x[6]+ c2*x[7]

X[3]=c3*x[0]+(-c7)*x[1]+(-c1)*x[2]+(-c5)*x[3]+c5*x[4]+ c1*x[5]+ c7*x[6]+ (-c3)*x[7]

X[4]=c4*x[0]+(-c4)*x[1]+(-c4)*x[2]+c4*x[3]+c4*x[4]+ (-c4)*x[5]+ (-c4)*x[6]+ c4*x[7]

X[5]=c5*x[0]+(-c1)*x[1]+c7*x[2]+c3*x[3]+(-c3)*x[4]+ (-c7)*x[5]+ c1*x[6]+ (-c5)*x[7]

X[6]=c6*x[0]+(-c2)*x[1]+c2*x[2]+(-c6)*x[3]+(-c6)*x[4]+ c2*x[5]+ (-c2)*x[6]+ c6*x[7]

X[7]=c7*x[0]+(-c5)*x[1]+c3*x[2]+(-c1)*x[3]+c1*x[4]+ (-c3)*x[5]+ c5*x[6]+ (-c7)*x[7]

Generic equation of 8-Point DCT to compute 1st sample is:

X[0]=k1*x[0]+ k2*x[1]+ k3*x[2]+ k4*x[3]+ k5*x[4]+ k6*x[5]+ k7*x[6]+ k8*x[7]

(2)

The equation of 8-Point forward DCT consists of 8 multiplication operations and 7 addition operations. The equivalent DFG of 8-Point forward DCT is shown in Fig. 3, where primary inputs are two types, k1 to k8 indicates DCT coefficients and x[0] to x[7] indicates input signals primary inputs, blue nodes indicate multiplication operation and green nodes indicate addition operations, corresponding operation/node number are mentioned using integer value (1 to 15) and finally the output is shown in a red box. It can be observed that all the output signal X[0] to X[7] can be computed by changing the k1 to k8 value.

The equivalent text file "**DCT_8pt.txt**" created from the data flow graph of 8 point DCT is shown in Fig. 4 below. This has been included in the tool download folder.

The entire DFG can be typed in a single line in a text file (notepad) without pressing enter button.
Here the format for every operation is: **operator type, input 1, input 2, node #**

For example for opn 1:
*, 0, 0, 1

0 – indicates independent input

```
*,0,0,1,*,0,0,2,*,0,0,3,*,0,0,4,*,0,0,5,*,0,0,6,*,0,0,7,*,0,0,8,+,1,2,9,+,3,9,10,+,4
,10,11,+,5,11,12,+,6,12,13,+,7,13,14,+,8,14,15
```

Fig.4 Input file of 8-point DCT in .txt format